About the Role of the Digital Technologies in the Education in Mathematics

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Abstract. The purpose of this article is to observe and compare the applications of some digital technologies in the mathematical education. The research is limited to two different types of approach. The first is to support the presenting of the teaching materials and the second is to use digital models by means of accessible programming tools. The scope includes some of the risks when applying digital technologies in the education. The design of the paper and the methods used are reader friendly, clear and comprehensible. One of the important conclusions, among others, is that digital models should be used at the final stage of the learning process. The original results can be implicated in practice immediately.

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1. Introduction

The Recommendation of the Council of Europe from 22 May 2018 determines eight key competences for lifelong learning. One of them is mathematical competence and competence in science, technology and engineering (The Council of the European Union, 2018). The document underlines the insufficient abilities of sufficient reading, mathematics, science and digital skills. The Council stresses that all competences "are all considered equally important and each of them contributes to a successful life in society". A variety of learning approaches and environments can be applied for increasing motivation of learners and improve their competency in mathematics.

All actors involved in learning process (learners, teachers, administrative staff) can be encouraged to use digital technologies to improve learning. This also applies to mathematics because it is very closely related to informatics. At the same time this would develop digital competences. In many cases the desired competences in mathematics directly correspond to and even coincide with the competences of informatics. You should look for opportunities to combine parts of the learning material in the two disciplines. Such type of integration can be quite useful (Garov, Georgieva, Kovacheva & Angelov, 2017).

This article treats some possibilities for applying digital tools in traditional, face-to-face math training. There are currently many IT products that can support the training process. No additional financial costs are needed. It is possible to use the available, well-known software, such as MS Office. In addition, there are many free products that can be easily applied in the training process. Even simply listing them is an impossible task. So, as the main focus is on the methodology of the use of digital technologies, the possibilities of the specific IT products will not be considered in detail.

Two main directions for the application of digital technologies in teaching mathematical subjects will be considered. The first one is to support the learning process in an illustrative way. The other one is related to a wider use of the computational capabilities of modern computing.

2. Computer-assisted presentations

The presentation of the material taught is a field for the widespread application of digital technologies in training. "Presentation" must be interpreted as presentation of learning content in any form – written, audio or video and in any way – face-to-face, distance or blended learning. In this case, different audio, video and text materials can be used, as well as well-structured presentations prepared with specialized software that enables related presentation of graphic, text, audio and video objects. Some of these software tools have advanced navigation capabilities, thus allowing the development of whole separate mini courses. Widely available and easy to comprehend are, for example, MS Power Point or Open Office Impress. Of course, these tools are applicable not only in teaching mathematical disciplines.

The advantage of using computer-based presentations is obvious. They help for a more attractive and effective presenting of the learning content. The presentation of the graphic images is much more accurate. There is also a possibility to return easily to a previous part of learning content.

However, the excessive use of such materials may not increase the quality of teaching. The amount of information must be tailored to the capabilities of the learners, i.e. the volume of the computer presentation (the number of slides) and the nature of the content must be adjusted to be accepted for the time of the lesson. Kawasaki's "10-20-30" rule is well-known, i.e. 10 slides to be presented in 20 minutes and the font of the text to be at least 30 pts[1]. Duarte cites expert opinions that the pace can be 1-2 slides per minute and shares her experience that a 40-minute presentation must not contain more than 100 slides[2]. It should be considered, however, that this is the volume and rate in a corporate presentation. A study found that when presenting learning content, the average number of slides used per 90-minute session was 22.7 with an average of 27.8 words per slide (Brock & Joglekar, 2011).

Another "danger" when using computer presentations is cluttering up the slides with effects that distract learners from the topic of the lecture and have a rather negative impact on the effectiveness of the learning process (Bartsch & Cobern, 2003).

Last but not least, there is a certain risk of losing the contact with the audience. Hendry cites opinions from professors from Carleton University, Canada, that teaching the classic "chalk and talk" method is preferable[3]. According to one of them, Professor Simon Power: "There is a huge advantage of chalk and talk if you're teaching math or economics. You can look around the class and if you see confusion or people not being happy, it's easy if you have a blackboard to explain a rule of calculus. A special attention should be paid to the quote by Professor Pam Woolff: "If I'm showing my students information that has been pre-printed on a slide, it's like I'm handing them information down from high. If I'm writing and talking and inviting them to talk about it, then it's something we developed together". Of course, instead of the classic blackboard, can be used, for example, tablet and pen-like stylus connected to a video projector (Vasilev & Milusheva, 2017).

Research about student preferences is too controversial. Some authors point out that students prefer traditional "chalk and talk" teaching over computer presentations (Saritha & Ramesh, 2018) and (Krishna, Datta, Kishan, & Bhanuprakash, 2012). The same thesis is based on the article "Rock the Chalk: A Five-Year Comparative Analysis of a Large Microbiology Lecture Course Reveals Improved Outcomes of Chalk-Talk Compared to PowerPoint" by Christopher M. Waters, Michigan State University (https://www.biorxiv.org/content/10.1101/644567v1, accessed June, 6, 2022). The majority of comments to the article support it. In other studies, however, the opposite thesis is supported – that lectures in the form of Power Point Presentation (Seth, Upadhyaya, Ahmad, & Moghe, 2010) are preferred.

The contradictory results of the studies cited lead to the conclusion that both methods have a lot of positive and negative aspects, and the presence or absence of computer presentations is not crucial. It is clear, that it all depends on the lecturer's claim. Excessive application of computer technology can be degenerate in a demonstration of the possibilities of the software. The use of many auxiliary materials by the lecturer may cause a negative reaction in the learners. They may be under the impression that the teacher is using computer files as a way to disguise his or her lack of competence. On the other hand, the "chalk and talk" method can create a barrier between the speaker and the audience. In my opinion, confidence and respect among students have teachers who are able to combine the "chalk and talk" method with continuous contact with the audience, using modern digital technologies only in accordance with the objectives of the training.

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3. Computational mathematical models

One of the earliest mentioning of the science mathematics, dating back to 5-th century BC, was: "Yes, and numbers, too, chiefest of sciences, I invented for them..."[4]. On the other hand, the word computer comes from compute. There are hardly any closer relatives than Aeschylus's modern heiress, the "chiefest of sciences", whose subject is the numbers and computer science that are the basis of digital technologies. Therefore, computer-based computational and simulation models may be and should be applied in the teaching of mathematical disciplines.

One of the possible criteria classifies the models as deterministic and probabilistic. Deterministic are those models in which all parameters are fully defined. Probabilistic models are those in which all the parameters (or some of them) can take random values. All these models can be presented using modern computers.

Applying deterministic models is easy even without special programming skills. The well-known Microsoft Excel is a suitable tool for quick calculation. The results can be presented in tabular and graphical form, which is convenient for examining dependencies and trends (Penev, 2014). Other free tools are also known to be used in training (Grozdev & Dekov, 2013). Software tools allow you to solve a larger number of examples within an academic hour, especially for tasks in which cyclical algorithms are applied, such as the linear optimization task (Milkova & Yordanova, 2014).

Software tools are very useful when presenting probabilistic models. The accessibility of computer equipment makes it possible to create simple simulation models, which by other means is impossible within the usually insufficient academic hours. Nowadays, there is hardly a programming language or general-purpose software that does not have built-in functions to generate random numbers. With their help, for example, you can experimentally check a distribution of functions of random dimensions (Mihaylov, 2019a). The exemplary scenario of such an experimental test may contain two stages. In the first of these, a theoretical conclusion is drawn on the distribution of the random function, and in the second, this theoretical distribution is modelled by a computer simulation.

MS Excel also has such capabilities. It describes, for example, simulation modeling of a matrix game using the built-in RAND function (Mihaylov, 2019b). The modelling is within a pre-prepared scenario that provokes the interest of students and has a positive effect on the comprehension of the learning material. It should be stressed on the fact that MS Excel's user interface presents the computational process in full and is thus suitable to be used in training.

The modeling of the matrix game is based on the uniform distribution. It is possible to model other distributions, but this can be done using the famous inverse transform sampling (Mihaylov, 2016). This indicates that a variety of probability models can be built using MS Excel.

For sure, the use of computer applications is not a mandatory condition when teaching mathematical disciplines. There are, however, many topics where their presence leads to
simplification. Some of them are:

1. Affine transformation.

Let a point in the space is presented by $M = (x, y, 1)$ and its image represented by:

$M' = (x', y', 1) = M \cdot A$, where $A = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix}$ is the transformation matrix.

Obviously, if computer modelling is not used, it is possible to demonstrate only basic transformations (symmetry, rotation, translation) within the academic hour with a maximum of two to three points. When using MS Excel, for example, the following model can be described:

Polygon (not necessarily convex) is set with the coordinates of its vertices in a matrix form. By multiplication with the transformation matrix, an image is obtained, with all the results displayed on a chart.

2. Approximation by the method of the least squares.

Let the dots (x_i, y_i) $1 \le i \le m$ are given tabularly.

If the problem is solved by hand, we are usually limited to approximation with a polynomial of a maximum second grade and the number of approximated points not exceeding 10. If MS Excel is used, it is possible that the approximating function is of the kind $F(x) = a_0 + a_1\varphi_1(x) + \ldots a_n\varphi_n(x)$, where $\varphi_j(x)$ are random elementary linearly independent functions (Nikolaev & Milkova, 2021). Then it is easy to compose the matrices:

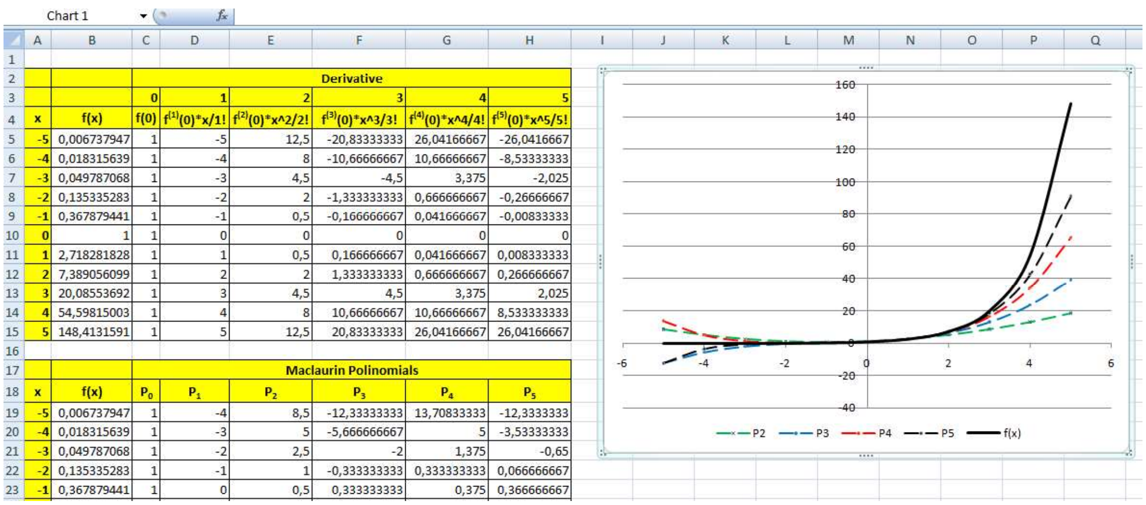
$X = \begin{pmatrix} 1 & \varphi_1(x_1) & \ldots & \varphi_n(x_1) \\ 1 & \varphi_1(x_2) & \ldots & \varphi_n(x_2) \\ \ldots & \ldots & \ldots & \ldots \\ 1 & \varphi_1(x_m) & \ldots & \varphi_n(x_m) \end{pmatrix}$ and $Y = \begin{pmatrix} y_1 \\ y_2 \\ \ldots \\ y_m \end{pmatrix}$, and from the matrix equation:

$X^T \cdot X \cdot A = X^T \cdot Y$ to find the vector $A = \begin{pmatrix} a_0 \\ a_1 \\ \ldots \\ a_n \end{pmatrix}$.

When using MS Excel, the number of points can be in the order of even hundreds, i.e. it is possible to approximate real data. This means that the principle of interdisciplinary approach can be extended to areas beyond mathematics and informatics.

3. Maclaurin polynomials.

Let the function $f(x)$ is given. Assume that its approximation is needed by a Maclaurin polynomial from second, third, fourth power, etc. The theoretical presentation of the material and the construction of polynomials does not give a sufficiently visual picture of the accuracy of the approximation. An exemplary model for approximation of the exponential function using MS Excel is given on Figure 1. You can compare the graph of a function $f(x)=e^x$ (the dense black line) with the graphs of McLaren's second to fifth degree polynomials (the broken lines).



		Derivative					
		0	1	2	3	4	5
x	f(x)	f(0)	$f^{(1)}$(0)*x/1!	$f^{(2)}$(0)*x^2/2!	$f^{(3)}$(0)*x^3/3!	$f^{(4)}$(0)*x^4/4!	$f^{(5)}$(0)*x^5/5!
-5	0,006737947	1	-5	12,5	-20,83333333	26,04166667	-26,0416667
-4	0,018315639	1	-4	8	-10,66666667	10,66666667	-8,53333333
-3	0,049787068	1	-3	4,5	-4,5	3,375	-2,025
-2	0,135335283	1	-2	2	-1,333333333	0,666666667	-0,26666667
-1	0,367879441	1	-1	0,5	-0,166666667	0,041666667	-0,00833333
0	1	1	0	0	0	0	0
1	2,718281828	1	1	0,5	0,166666667	0,041666667	0,008333333
2	7,389056099	1	2	2	1,333333333	0,666666667	0,266666667
3	20,08553692	1	3	4,5	4,5	3,375	2,025
4	54,59815003	1	4	8	10,66666667	10,66666667	8,533333333
5	148,4131591	1	5	12,5	20,83333333	26,04166667	26,04166667

		Maclaurin Polinomials					
x	f(x)	P_0	P_1	P_2	P_3	P_4	P_5
-5	0,006737947	1	-4	8,5	-12,33333333	13,70833333	-12,3333333
-4	0,018315639	1	-3	5	-5,666666667	5	-3,53333333
-3	0,049787068	1	-2	2,5	-2	1,375	-0,65
-2	0,135335283	1	-1	1	-0,333333333	0,333333333	0,066666667
-1	0,367879441	1	0	0,5	0,333333333	0,375	0,366666667

Figure 1. Approximation of e^x by Maclaurin polynomials.
Source: Screenshot from MS Excel. Own elaboration

The application of computer-based models in these cases leads to the deepening of knowledge and improvement of the skills of the learners. In the process of learning mathematics, elements of an experiment are embedded. It should be stressed that a scientific experiment cannot be expected to be carried out within the course of the school hour to discover new results (Grozdev & Lazarov, 2013). Of course, there are examples of mathematical discoveries arising from experimental application of digital technologies, but work on them only begins within the learning process and then continues in additional forms, for years. As an example here we may point out the problem for determining the dilation factor of the Peano-Hilbert curve (Rusakov, 2016). However, such cases are rare and the purpose of applying an experiment in the learning process should not be so ambitious. Digital models should be applied to confirm theoretical knowledge in mathematics as, for example, in physics class using an elementary model of a mathematical pendulum, the well-known value of earth acceleration is determined. However, such experiments can make the learning material more attractive and increase the interest of the students.

The examples mentioned above do not, of course, exhaust the topic. Each teacher, depending on the material being taught, their experience and the audience, should be able to create appropriate digital models.

One possible scenario for integrating a digital model into mathematics training may have the following sequence:

Step 1. Presentation of theoretical knowledge (theorem, method, algorithm).

Step 2. Solving an example that supports the theory in a traditional way.

Step 3. Solving examples with the application of digital technologies and study the solution when changing the parameters of the model.

The direct application of digital technologies has some risks. An experiment is well-known – two groups of students were asked if the function $f(x)=\ln x+10\sin x$ has an infinite limit as x is approaching $+\infty$. Half of the students used graphical calculators and the rest – the traditional paper-and-pencil resources. If you answer the question applying the theorems for limits, the positive answer comes quite quickly. However, the following phenomenon occurred: in the graphic calculator environment, 25% of students answered no, appealing to the oscillation of the observed graphic representation; in the paper-and-pencil environment, only 5% of students answered no (Trouche, 2005). It can therefore be concluded that applying digital technologies without having the skills to work on the paper-and-pencil method does not have a positive effect. Therefore, Step 2 of the scenario proposed above should have been deemed as mandatory.

4. Conclusion

In the recent decades, digital technologies have entered all sectors of society and economy. They caused very profound changes in technology and social relations in life. Objectively, they also need to be applied in the "knowledge economy" by changing the forms of provision of learning content.

Linking math teaching to digital technologies is a manifestation of cross-discipline learning and can enrich teaching. An even better effect would be achieved if elements of other subjects were integrated into the mathematics curriculum. This can be a promising direction of work.

Last but not least, it should be stressed that the role of digital technologies in training is important, but not the main one. The degree of their application must correspond to the material taught and to the level of the learners. They are just a tool and cannot replace the teacher's activity. His preparation, skills and motivation are crucial for the successful achievement of the training objectives.

5. Notes

1. Guy Kawasaki, an American marketing specialist, author of the concepts of "Evangelism Marketing" https://guykawasaki.com/the_102030_rule/.
2. According to Duarte Inc's website, a company providing presentation creation services. https://www.duarte.com/use-the-right-number-of-presentation-slides/.
3. Hendry, Cassandra. *Chalkboard teaching in the age of technology*, https://carleton.ca/edc/wp-content/uploads/2015-16-Reflections-Newsletter.pdf
4. Aeschylus. *Prometheus Bound*, Verse 459. Translated by Herbert Weir Smyth.

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